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Published online 2015 February 20. **Research Article**

A Two-Stage Stochastic Programming for Nurse Scheduling in Razavi Hospital

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Background: One of most important operations research problems is Nurse Scheduling Problem (NSP) that tries to find an optimal way to assign nurses to shifts with a set of hard constraints. Most of the researches are dealing with this problem in deterministic environment with constant parameters. While In the real world applications of NSP, the stochastic nature of some parameters like number of arriving patients, stay periods, etc. are some sources of uncertainties that need to be controlled to provide a qualified schedule.
Objectives: In this article we propose our model in an uncertain environment in Department of Heart Surgery in Razavi Hospital.
Materials and Methods: The demand and stay period of patients are stochastic whose the distribution is determined from historical data. Finally, the demand of nurses in each shift in planning horizon is calculated regarding the priority (vitality) of patients.
Results: The stochastic optimization is adapted for our problem and the Sample Average Approximation (SAA) method is used to obtain a optimal schedule with respect to minimizing the regular and over time assignment costs. The results have been analyzed and show the validity of our model.
Conclusions: In this model the demand and stay period of patients are stochastic whose the distribution is determined from historical data. We also consider the priority (vitality) of patients in our model.

Keywords: Nurse; Uncertainty; Stochastic Processes; Patients

1. Background

In the NSP, different constraints with different importance should be considered. Constraints can be classified into two groups: Hard and Soft constraints, which strongly depend on individual preferences, system preferences and government regulations. The hard constraints must be satisfied to achieve feasible solutions that include demand coverage requirements, while soft constraints are desirable but not necessary, and thus can be violated. The NSP has several objective functions such as maximizing nurse preferences, minimizing number of nurse in work shifts, minimizing nurse assignment costs, etc. Due to different approaches to solving the NSP, The used approaches can be classified into three groups: optimization approaches, artificial intelligence approaches and heuristic and meta-heuristic approaches. In the following, an overview of the researches in this field will be discussed.

1.1. Different Approaches in Nurse Scheduling Problem

1.1.1. Optimization Approaches [Mathematical Programming]

In the early 1970s, Warner (1) presented a multiple option programming to the NSP, that in this research each nurse describe a group of variables and each variable within a group is a possible schedule for that nurse. Millar et al. (2) provided a mathematical model for cyclic and non-cyclic type for NSP. The main objective is minimizing nurse assignment costs. Venkataraman (3) presented a nurse scheduling system for evaluating nurse preference management regulations. For this purpose, a mixed-integer liner programming is used to specify nurse preferences for planning horizon. Ozkarahan (4) proposed a flexible decision support system that will satisfy the preferences of both hospitals and nurses that attempt to improve flexible work pattern in scheduling problems. Jaumard et al. (5) presented a generic 0-1 linear programming model for a complex NSP, which considers most of a real situation condition. The main objective is minimizing salary costs and maximizing both nurse preferences and team balance to satisfy the demand coverage constraints. Klinz et al. (6) used two mathematical models for a type of the NSP in order to minimize the total number of work shifts. Bard et al. (7) proposed an integer programming model to produce most efficient nurse scheduling system for regular and pool nurses in different conditions to satisfy expected demand in planning horizon. Hattori et al. (8) presented a nurse scheduling system based of Constraint Satisfaction Problem (CSP) that constraints have different levels of importance and can change dynamically. Parr et al. (9) used Sawing and Noising with simulated annealing in NSP to ensure that nurses on each shift are sufficient. Another study (10) proposed a stochastic integer programming model for NSP in order to minimize workload penalty on nurses and a Benders' decomposition approach is applied to solve this problem. A greedy algorithm is proposed to solve the recourse sub problems. Fan et al. (11) used binary integer linear programming to formulate the NSP in order to maximize the satisfactions nurses and hospital regulations.

1.1.2. Artificial Intelligence Approaches

Li et al. (12) used the Bayesian optimization and classifier systems for NSP to minimize total preference cost of nurses. Topaloglu et al. (13) presented a fuzzy goal programming model for NSP to measure uncertainty in objective value of hospital regulations, nurse preferences and constraints. Topaloglu et al. (14) proposed a multi-objective integer programming for NSP to treat and to produce an equitable schedule for nurses, and to satisfy for hospital management preferences.

1.1.3. Heuristic and Meta-Heuristic Approaches

Maenhout et al. (15) presented a novel electromagnetism meta-heuristic technique for the NSP to minimize the total pattern penalty cost. Landa silva, et al. (16) used a multi-objective approach to cope with a real-world condition in NSP. To this end, used an evolutionary algorithm to achieve a good quality non-dominated schedules so that the scheduler can choose the most appropriate one for available nurses. Tsai et al. (17) presented a two-stage mathematical modeling for a NSP with respect to hospital management requirements, government regulations, and nurse preferences. Ohki et al. (18) used a new approach by using Cooperative Genetic Algorithm (CGA) to solve NSP. Zhang et al. (19) proposed a hybrid Swarm-based optimization algorithm in hospital environments that incorporates Genetic algorithm and variable neighborhood search to cope highly-constrained NSP. In the real world applications of NSP, vagueness of information on objective values of management objectives and nurse preferences are some sources of uncertainties that need to be managed in providing a good quality schedule. For this purpose, the basic parameters such as the demand and patient stay period are stochastic and the distribution of these parameters is determined from historical data. The demand for nurses in each shift is determined by considering the priority (vitality) of patients that are present in the shift. The rest of this article is as follows: in Section 2, we present the proposed optimization model for NSP and the structure of the model is investigated. In Section 3, the solution approach is introduced with detailed description of SAA method. Numerical experiments are presented in Section 4. Finally, concluding remarks are described in Section 5.

1.2. The Model for Nurse Scheduling Problem

The indices, parameters and variables are summarized in Table 1, and in the following our mathematical model describes Stochastic Nurse Scheduling Problem (SNSP).

Table 1. Notation Summary

Indices and Sets	
i ϵI	Index of available nurses i = 1, 2,. . ., 18
j ϵJ	Index of shift j = 1, 2, 3
kϵZjt	Index of patient k = 1, 2, . . ., Z_{jt}
tϵT	Index of date in planning horizon t = 1, 2, . . ., T
ξϵB	Index of scenario ξ = 1, 2, . . ., B
TA	Index of allowed dates for assigning nurse 1 and 2, TA ϵ {1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24, 25, 27, 28, 29, 30, 31}
Parameters	
pj	Normal cost of nurse assignment in shift j
Oj	Overtime cost of nurse assignment in shift j
Q	Total number of beds in each shift
Qjt	Empty capacity of shift j in date t
Zjt	Number of new patient arriving in shift j in date t
Ajt	Maximum number of allowed patients in shift j in date t
Nkjt	Number of shifts that patient k in date t (entered in shift j) remains in the system
βmjt	Priority (vitality) of patient m present at shift j in date t β_{mjt} ϵ {0.25,0.50,0.75,1.00}
Pw, k, j-w	A binary variable that indicates whether patient k (presented shift w) is still at shift j or not.
Tjt	Total number of patients in shift j in date t
Variables	
xijt	1 if nurse i is assigned to shift j in date t, 0 otherwise
Vjt	Additional nurses in shift j in date t
Rjt	Demand for nurses in shift j in date t

$$\mathrm{MINz} = \sum_{i=1}^{18}\sum_{j=1}^{3}\sum_{t=1}^{31} p_j x_{\mathrm{ijt}} + \sum_{\xi\in B}\sum_{j=1}^{3}\sum_{t=1}^{31}\varphi(\xi) O_j V_{\mathrm{jt}}^{\xi},$$

St:

$X_{i1t} = 1 \; i = 1, 2 \quad , t \in T_A$ (1)

$x_{i2t} + x_{i3t} \leq 1 \quad \forall i = 3, \ldots, 18, t = 1,2,\ldots,31$ (2)

$x_{i3t} + x_{ij(t+1)} \leq 1 \quad \forall i = 3,\ldots,18, t = 1, 2,\ldots, 31, j = 1, 2, 3$ (3)

$$\sum_{j=1}^{2}\sum_{t=1}^{31} x_{\mathrm{ijt}} + 2\sum_{t=1}^{31} x_{i3t} \geq 26$$

i = 1, 2, . . ., 18 (4)

$$Q_{\mathrm{jt}}^{\xi} = Q - \sum_{w=1}^{j-1}\sum_{k=1}^{(Z_{j-w})^{\xi}} \left(p_{w,k,j-w}\right)^{\xi}$$

t = 1, 2, . . ., 31, j = 1, 2, 3, ξ = 1, 2, . . ., B (5)

$$Z_{jt}^{\xi} \leq Q_{jt}^{\xi} + \left(1 - h_{jt}^{\xi}\right) M$$

t = 1, 2, …, 31, j = 1, 2, 3,ξ = 1, 2, …, B (6)

$$Z_{jt}^{\xi} > Q_{jt}^{\xi} - \left(1 - d_{jt}^{\xi}\right) M$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (7)

$$d_{jt}^{\xi} + h_{jt}^{\xi} = 1$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (8)

$$A_{jt}^{\xi} = Z_{jt}^{\xi} h_{jjt}^{\xi} + Q_{jt}^{\xi} d_{jt}^{\xi}$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (9)

$$T_{jt}^{\xi} = A_{jt}^{\xi} + \sum_{w=1}^{j-1} \sum_{k=1}^{(Z_{j-w})^{\xi}} \left(p_{w,k,j-w}\right)^{\xi}$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (10)

$$R_{jt}^{\xi} \leq \sum_{m=1}^{T_{jt}^{\xi}} \left(\beta_{\mathrm{mjt}}\right)^{\xi}$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (11)

$$R_{jt}^{\xi} > \sum_{m=1}^{T_{jt}^{\xi}} \left(\beta_{\mathrm{mjt}}\right)^{\xi} + 1$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (12)

$$R_{jt}^{\xi} > \sum_{i=1}^{18} x_{ijt} - \left(1 - f_{jt}^{\xi}\right)$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (13)

$$R_{jt}^{\xi} \leq \sum_{i=1}^{18} x_{ijt} + \mathrm{M} f_{jt}^{\xi}$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (14)

$$V_{jt}^{\xi} = \left(R_{jt}^{\xi} - \sum_{i=1}^{18} x_{ijt}\right) f_{jt}^{\xi}$$

t = 1, 2, …, 31, j = 1, 2, 3, ξ = 1, 2, …, B (15)

$$f_{j}^{\xi}, d_{j}^{\xi}, h_{j}^{\xi}$$

ϵ (0, 1) M is a big numberj = 1,2,3, ξ=1,2,. . .,B (16)

Objective is minimizing the regular and over time assignment costs. Let φ (ξ) be the corresponding probability of scenario ξ = 1, 2, 3, …, B and $\sum_{\xi 1013;B} \varphi(\xi) = 1$ (1) Assures that nurse 1 and 2 are assigned to shift one in allowed dates. Based on hospital regulations, the nurse 1 and 2 (head nurses) should be assign to shift one (morning shift) in working days. In this hospital no one is allowed to work on two consecutive afternoon and night shifts. (2) Applies this constraint. If a nurse is assigned to a night shift, he/she is no allowed to work in the following days. (3) Consider this limitation. (4) shows that every nurse should work at least 26 shifts, knowing that Shift 3 (night shift) has double work load in compression with shift 1 and 2 (morning and afternoon shifts). (5) Shows how empty beds in shift j in date t can be calculated from total beds available and number of patients that are present in shift j in date t. (6-9) calculate the number of accepted patients in shift j in date t considering the remaining capacity. (10) Shows total number of patients present in shift j in date t. (11-12) determine the (integer) value of demand in shift j date t by taking into account the priority factor of total patient in shift j. (13-15) are calculating additional nurses in shift j in date t. For this purpose, if number of existing nurses are less than required ones, the number of over time nurses will be a positive value, calculate by (15).

In this research Z_{jt} and N_{kjt} are assumed to be uniformly (discrete) distributed: $Z_{jt} \sim DU[a, b]$ and $N_{kjt} \sim DU[c, d]$ for k = 1, 2, 3, …, Z_{jt}, t = 1, 2, …, 31, j = 1, 2, 3.

An exact solution can be achieved by enumeration for a small size problem. However, if the problem size gets bigger, the model is unmanageable. This article employs Sample Average Approximation (SAA) algorithm as a solution strategy for Stochastic Nurse Scheduling Problem. For applying the SAA algorithm, the SNSP is reformulated using recourse action model and the basic properties of the new formulation are investigated.

1.2.1. Stochastic Programming with Recourse Action

The most important group of stochastic programming models, known as recourse models, that is calculated by allowing recourse actions after realizations of the random variables (T, h). Given a first-stage decision x for all possible realization, (q, T, h) of (q, T, h). h-T (x) Are compensated at minimum costs by select second-stage decisions as an optimal solution of the second-stage problem.

min

y
qy
St:
Wy = h - Tx,
y ϵ Y

In second-stage problem, q is the recourse action unit cost vector and the recourse matrix W determines the available technology. We will use $\xi = (q, T, h)$ to denote characterize for all randomness in the problem. The objective function of this second-stage problem, determine the minimum recourse action costs as a function of the first-stage decision x and a realization of ξ, will be defined by $v(x, \xi)$; its expectation $Q(x) = E_{\xi}[v(x, \xi)]$ gives the expected recourse action costs associated with a first-stage decision x. Thus, the two-stage recourse action model is:

$min_x cx + Q(x)$

St:

$Ax = b$

$x \in X$

Where, $cx + Q(x)$ specifies the total expected costs of a decision x, Stougie, et al. (20). The problem SNSP can be formulated using the following recourse model.

$$(\text{SNSP1})\, \text{MINz} = \sum_{i=1}^{18}\sum_{j=1}^{3}\sum_{t=1}^{31} p_j x_{ijt} + E\left[Q(x,\xi)\right],$$

S t :

$X_{i1t} = 1$

$i = 1, 2$, $t \in T_A$ (1)

$x_{i2t} + x_{i3t} \le 1$ $\forall I = 3, \ldots, 18, t \in T$ (2)

$x_{i3t} + x_{(ij(t+1))} \le 1$ $\forall i = 3, \ldots, 18, t \in T, j = 1, 2, 3$ (3)

$$\sum_{j=1}^{2}\sum_{t=1}^{31} x_{ijt} + 2\sum_{t=1}^{31} x_{i3t} \ge 26$$

$I = 1, 2, \ldots, 18$ (4)

$x_{ijt} \in \{0,1\}$ $i = 1, 2, \ldots, 18, j = 1, 2, 3, t = 1, 2, \ldots, 31$

Where E [Q(x, ξ)] is the recourse action function, and

$$(\text{SNSP2})\, Q(x,\xi)\, \text{MIN} = \sum_{\xi \in B}\sum_{j=1}^{3}\sum_{t=1}^{31} \varphi(\xi) Q_j V_{jt}^{\xi},$$

St:

$$Q_{jt}^{\xi} = Q - \sum_{w=1}^{j-1}\sum_{k=1}^{(Z_{j-w})^{\xi}} \left(p_{w,k,j-w}\right)^{\xi}$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (5)

$$Z_{jt}^{\xi} \le Q_{jt}^{\xi} + \left(1 - h_{jt}^{\xi}\right) M$$

$t = 1, 2, \ldots, 31$ $j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (6)

$$Z_{jt}^{\xi} > h_{jt}^{\xi} - \left(1 - d_{jt}^{\xi}\right) M$$

$t = 1, 2, \ldots, 31$ $j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (7)

$$d_{jt}^{\xi} + h_{jt}^{\xi} = 1$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1 2, \ldots, B$ (8)

$$A_{jt}^{\xi} = Z_{jt}^{\xi} h_{jt}^{\xi} + Q_{jt}^{\xi} d_{jt}^{\xi}$$

$t = 1, 2, \ldots, 31, j = 1 2, 3, \xi = 1, 2, \ldots, B$ (9)

$$T_{jt}^{\xi} = A_{jt}^{\xi} + \sum_{w=1}^{j-1}\sum_{k=1}^{(Z_{j-w})^{\xi}} \left(p_{w,kj-w}\right)^{\xi}$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (10)

$$R_{jt}^{\xi} \le \sum_{m=1}^{T_{jt}^{\xi}} \left(\beta_{mjt}\right)^{\xi}$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (11)

$$R_{jt}^{\xi} > \sum_{m=1}^{T_{jt}^{\xi}} \left(\beta_{mjt}\right)^{\xi} + 1$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (12)

$$R_{jt}^{\xi} > \sum_{i=1}^{18} x_{ijt} - \left(1 - f_{jt}^{\xi}\right) M$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (13)

$$R_{jt}^{\xi} \le \sum_{i=1}^{18} x_{ijt} + \text{M} f_{jt}^{\xi}$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (14)

$$V_{jt}^{\xi} = \left(R_{jt}^{\xi} - \sum_{i=1}^{18} x_{ijt}\right) f_{jt}^{\xi}$$

$t = 1, 2, \ldots, 31, j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (15)

$$f_j^{\xi} d_j^{\xi} h_j^{\xi}$$

$\in (0,1)$, M is a big number $j = 1, 2, 3, \xi = 1, 2, \ldots, B$ (16)

Evaluating the value of E [Q(x,ξ)] is very hard, because of the large random data vector ξ∈B. It involves solving a large number similar Integer Liner Programming (ILP). Birge, et al. (21). As, it is difficult to solve our model exactly, the following structural of the SNSP2 is proposed to provide an approximation.

2. Materials and Methods

Suppose that we can produce a sample $\xi^1, \xi^1, \ldots, \xi^N$ of N replications of the random vector ξ. So that each ξ^j, j = 1, …, N has the same probability distribution. We can approximate the value of expectation function q(x) = E [Q(x, ξ)] by the average

$$g_N(x) = \frac{1}{N}\sum_{j=1}^{N}\left(\text{x}, \xi^j\right)$$

and so the "true" (expectation) problem is equal to:

$$g_N(x) = C^T x + \frac{1}{N}\sum_{j=1}^{N} \text{Q}\left(\text{x}, \xi^j\right)$$

Shapiro, et al. (22). The following mathematical model explains the SAA problem of the SNSP with sample size N.

$$\text{MINz} = \sum_{i=1}^{18}\sum_{j=1}^{3}\sum_{t=1}^{31} p_j x_{ijt} + \frac{1}{N}\sum_{n=1}^{N}\sum_{j=1}^{3}\sum_{t=1}^{31} O_j V_{jt}^{n},$$

St:

$x_{i1t} = 1$ $i = 1, 2$, $t \in T_A$ (1)

$x_{i2t} + x_{i3t} \le 1$ $\forall I = 3, \ldots, 18, t \in T$ (2)

$x_{i3t} + x_{(ij(t+1))} \le 1$ $\forall I = 3, \ldots, 18, t \in T, j = 1, 2, 3$ (3)

$$\sum_{j=1}^{2}\sum_{t=1}^{31} x_{ijt} + 2\sum_{t=1}^{31} x_{i3t} \ge 26$$

$i = 1, 2, \ldots, 18$ (4)

$$Q_{\mathrm{jt}}^{n}=Q-\sum_{w=1}^{j-1}\sum_{k=1}^{(z_{j-1})^{n}}\left(p_{w,k,j-w}\right)^{n}$$

t = 1, 2, …, 31, j = 1, 2, 3, n = 1, 2, …, N (5)

$$Z_{\mathrm{jt}}^{n}\leq Q_{\mathrm{jt}}^{n}+\left(1-h_{\mathrm{jt}}^{n}\right)M$$

t = 1, 2, …, 31, j = 1, 2, 3, n = 1, 2, …, N (6)

$$Z_{\mathrm{jt}}^{n}>Q_{\mathrm{jt}}^{n}-\left(1-d_{\mathrm{jt}}^{n}\right)$$

t = 1, 2, …, 31, j = 1, 2, 3, n = 1, 2, …, N (7)

$$d_{\mathrm{jt}}^{n}+h_{\mathrm{jt}}^{n}=1$$

t = 1, 2, …, 31, j = 1, 2, 3, n = 1, 2, …, N (8)

$$A_{\mathrm{jt}}^{n}=Z_{\mathrm{jt}}^{n}h_{\mathrm{jt}}^{n}+Q_{\mathrm{jt}}^{n}d_{\mathrm{jt}}^{n}$$

t = 1, 2, …, 31, j = 1, 2, 3, n = 1, 2, …, N (9)

$$T_{\mathrm{jt}}^{n}=A_{\mathrm{jt}}^{\xi}+\sum_{w=1}^{j-1}\sum_{k=1}^{(z_{j-w})^{n}}\left(p_{w,k,j-w}\right)^{n}$$

t = 1, 2, …, 31, j = 1, 2, 3, n = 1, 2, …, N (10)

$$R_{\mathrm{jt}}^{n}\leq\sum_{m=1}^{T_{\mathrm{jt}}^{n}}\left(\beta_{\mathrm{mjt}}\right)^{n}$$

t = 1, 2,…, 31, j = 1, 2, 3, n = 1, 2, …, N (11)

$$R_{\mathrm{jt}}^{\xi}>\sum_{m=1}^{T_{\mathrm{jt}}^{n}}\left(\beta_{\mathrm{mjt}}\right)^{n}+1$$

t = 1, 2,…, 31, j = 1, 2, 3, n = 1, 2, …, N (12)

$$R_{\mathrm{jt}}^{n}>\sum_{i=1}^{18}x_{\mathrm{ijt}}-\left(1-f_{\mathrm{jt}}^{n}\right)M$$

t = 1, 2,…, 31, j = 1, 2, 3, n = 1, 2, …, N (13)

$$R_{\mathrm{jt}}^{n}=\sum_{i=1}^{18}x_{\mathrm{ijt}}+\mathrm{M}f_{\mathrm{jt}}^{n}$$

t = 1, 2,…, 31, j = 1, 2, 3, n = 1, 2, …, N (14)

$$V_{\mathrm{jt}}^{n}=\left(R_{\mathrm{jt}}^{n}-\sum_{i=1}^{18}x_{\mathrm{ijt}}\right)f_{\mathrm{jt}}^{n}$$

t = 1, 2,…, 31, j = 1, 2, 3, n = 1, 2, …, N (15)

$$f_{j}^{n},d_{j}^{n},h_{j}^{n}$$

∊ (0, 1), M is a big number j = 1, 2, 3, n = 1, 2, …, N (16)

Kleywegt et al. (23) and Shapiro et al. (24, 25) proposed a general SAA algorithm for a kind of Stochastic Discrete Optimization Problem (SDOP). The procedure is as follows:

Suppose M be the number of replications in sample, N be the number of scenarios in the sampled problem, and N´ be the number of sample used to estimate $C^{T}\hat{x}+\mathrm{E}\left[Q\left(\hat{X},\xi\right)\right]$ for a given feasible solution X.

1. For $m=1\ldots M$ do step 1.1 through 1.3

1.1. Generate N sample $\xi^{1},\ldots,\xi^{N}$

1.2. Solve the proposed SAA problem and $\bar{z}_{N}^{m}$ let $\bar{X}_{N}^{m}$ and for optimal objective function value and optimal solution.

1.3. Generate $\overset{769;}{N}$ independent random sample and evaluate the objective function value $g_{\overset{769;}{N}}\left(\bar{X}\right)$ and variance for feasible solution X. Different methods can be used for obtaining a good feasible solution $\bar{X}$. One good method is that a deterministic problem with expected value parameters, known the Expected Value Problem (EVP). Another good method is solving a problem without stochastic constraints. This method is used for a two-stage stochastic problem.

$$g_{\overset{769;}{N}}\left(\bar{X}\right)=\sum_{i=1}^{18}\sum_{j=1}^{3}\sum_{t=1}^{31}p_{j}\bar{X}_{\mathrm{ijt}}+\frac{1}{\overset{769;}{N}}\sum_{n=1}^{\overset{769;}{N}}\sum_{j=1}^{3}\sum_{t=1}^{31}O_{g}V_{\mathrm{jt}}^{n},$$

$$S^{2}_{g_{\overset{769;}{N}}\left(\bar{X}\right)}=\frac{1}{\overset{769;}{N}\left(\overset{769;}{N}-1\right)}\sum_{n=1}^{\overset{769;}{N}}\left[\sum_{i=1}^{18}\sum_{j=1}^{3}\sum_{t=1}^{31}p_{j}\bar{X}_{\mathrm{ijt}}+\sum_{j=1}^{3}\sum_{t=1}^{31}O_{j}V_{\mathrm{jt}}{}^{n}-g_{\overset{769;}{N}}\left(\bar{X}\right)\right],$$

2. Calculate $\bar{Z}_{N}^{M}$ and $S_{\bar{Z}_{N}^{M}}^{2}$.

$$\bar{Z}_{N}^{M}=\frac{1}{M}\sum_{m=1}^{M}Z_{N}^{m},S_{Z_{N}^{M}}^{2}=\frac{1}{(\mathrm{M}-1)\mathrm{M}}\sum_{m=1}^{M}\left[Z_{N}^{m}-Z_{N}^{m}\right]^{2},$$

3. For each solution X_{N}^{m}, m = 1… M calculate the optimality gap by $g_{\overset{769;}{N}}\left(X_{N}^{m}\right)-\bar{Z}_{N}^{m}$ and variance of. $S^{2}_{g_{N}\left(\bar{X}_{N}^{M}\right)}+S^{2}_{Z_{N}^{M}}$ Finally choose one of the M candidate solutions.

In the algorithm, we can obtain a lower bound (LB) and an upper bound (UB) to the true optimal value by estimating X_{N}^{m} and $g_{\overset{769;}{N}}\left(X_{N}^{m}\right)$ respectively, Mak et al (26). $\bar{Z}_{N}^{M}$ Is an unbiased estimator of optimal objective function $\mathrm{E}\left[Z_{N}\right]$ that $\bar{Z}_{N}^{M}=E\left[Z_{N}\right]\leq Z^{*}$. We have that $g_{\overset{769;}{N}}\left(X_{N}^{m}\right)$ is an unbiased estimator of the true objective value, but $\mathrm{E}\left[g_{\overset{769;}{N}}\left(X_{N}^{m}\right)\geq Z^{*}\right]$.

3. Results

3.1. Example Data

This section describes a case study in Department of Heart Surgery in Razavi Hospital. In this department, there are sixteen nurses (i = 3… 18), a head nurse and an assistant head nurse (i = 1, 2) and we have 31 days which in each day there are 3 shifts (Morning, Afternoon and Night). The system capacity (the available total beds) is 25 beds. In order to obtain the discrete distribution of number of new patient arriving in shift j in date t and number of shifts that patient k remains in system, the actual data of hospital is analyzed. The disruption of these stochastic parameters are estimated as: Z_{jt} ~ DU [3,7] and N_{kjt} ~ DU

[5,9] for k = 1, 2, 3, …, Z_{jt}, j = 1, 2, 3, t = 1, 2, …, 31. The regular and over time assignment costs for each shift are estimated 15$ and 18$, respectively.

3.2. Experimental Results

In this section, the experimental results achieved from the implementation of our approach in Department of Heart Surgery in Razavi Hospital are discussed. In our approach, we used N = [1-2-3-4-5-6-7-8-9-10-20-30-40-50-100], M = 20 and N’ = 10,000 for SAA algorithm presented in Section 3. Figure 1 demonstrates the SAA algorithm optimal solution changes relative to different sample sizes of N. Although the original sample space in SNSP model is extremely large, a high-quality solution can be obtained by a relatively small sample size. For the solution of numerical experiment, a sample size of 100 can provide an acceptable solution. The sample size (i.e. the size of scenario set B) in numerical experiments is calculated by the sample size of Z_j and N_{kj} which are the possible realization of the number of new patients arriving in shift j as well as the number of shifts that patient k remains in the system respectively. All experiments are not needed to evaluate the effects of variable sizes in our SNSP model. It is proven by Mak et al. (26), however, the sample size required to obtain optimal solutions and optimal values functions is logarithmical to the variable size. Therefore, the required sample size increases linearly proportional to a rise in the number of new patients arriving in shift j and number of shifts that patient k remains in system. The results concerning the achievement of optimal solutions and optimal values show the convergence of the SAA solutions with exponential rates. The EVP solution is obtained by substituting random parameters Z_j and N_{kj} by their mean values and then solving the deterministic problem. The results of solving SNSP model shows that the deterministic problem (EVP) yields unsatisfactory solutions. We can obtain a lower bound (LB) and an upper bound (UB) for true optimal solutions and optimal values as functions by estimating $\bar{z}_N^M$ ZNM and $g_{\acute{N}}(X_N^m)$ respectively. The former is an unbiased estimator of the optimal objective function $E[z_N]$ in which $z_N = E\lfloor z_N \rfloor$ In estimating the lower bound, we can generate M independent samples of the uncertain parameters, each with the size N, and by solving the corresponding SAA problems, the optimal objective values $Z_N^1, \dots, Z_N^M$ can be obtained Then, it can be estimated that: $z_N = \frac{1}{N}\sum_{m=1}^{M} Z_N^m$

It is an unbiased estimator of $E[z_N]$, and therefore a statistical lower bound to Z*. $g_{\acute{N}}(X_N^m)$ is an unbiased estimator of the true objective value $E\left[g_{\acute{N}}(X_N^m)\right] \geq Z^*$. In calculating $g_{\acute{N}}(X_N^m)$ each solution $X_N^m, m = 1 \dots M$ of random sample N´, the objective functions are evaluated and a statistical upper bound to the optimal value (Z*) of the stochastic programming with integer recourse is provided. For a fixed sample size, the proposed statistical and deterministic lower and upper bound techniques can be useful to validate the quality of a candidate optimal solution. In implementation of numerical experiments, the value of M (number of replications in the simulation procedure) is set to 20. Table 2 shows statistical lower and upper bounds, indicating that 20 replications suffice to obtain a reasonable confidence interval for statistical lower and upper bounds. If the variances of statistical lower and upper bounds are too large, the value of M should be increased. The assignment of nurses to optimal sequence in the planning horizon is demonstrated in Table 3. When assigning nurses to regular and overtime shifts, the proposed deterministic constraints should be considered. A summary of the simulation procedure and the gap calculation between stochastic nurse scheduling problem (SNSP) and expected value problem (EVP) is given in Figure 2. Arbitrarily, the run length is determined as 100 iterations. The results of solving SNSP model show that the solution presented by the deterministic problem (EVP) is less desirable than the one provided by the SAA problem, and there is an evident difference between SNSP and EVP solutions.

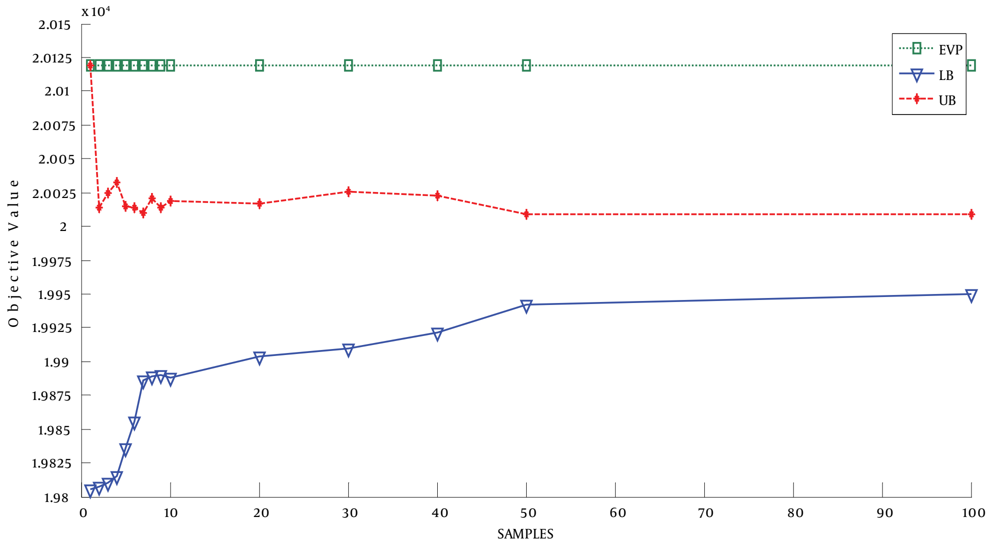

Figure 1. Optimal Solution Changes Relative to the Sample Size N

Table 2. Experimental Results

Sample size N	95% CI of LB	95% CI of UB	Optimally Gap
1	(19768.65772-19805.10600)	(20114.17679-20124.82321)	33.63927
2	(19771.29234-19807.59600)	(20009.18727-20019.28073)	32.97472
3	(19774.08897-19810.24842)	(20020.23775-20018.37825)	30.66832
4	(19779.58859-19815.60311)	(20029.16223-20011.74977)	27.78374
5	(19799.73321-19835.60332)	(20011.70872-20018.74328)	27.12840
6	(19819.87784-19855.42524)	(20010.99321-20017.33273)	25.68278
7	(19850.02246-19885.86774)	(20007.15968-20013.08832)	23.08402
8	(19853.19909-19888.00364)	(20017.71616-20013.95355)	23.00442
9	(19854.49571-19888.25367)	(20013.82265-20016.64535)	20.84923
10	(19853.11034-19888.80847)	(20016.89513-20021.16487)	18.79383
20	(19868.46904-19903.33432)	(20014.91161-20018.62839)	16.12119
30	(19875.01058-19909.86800)	(20021.17609-20027.33991)	13.78742
40	(19886.96421-19921.35353)	(20021.20657-20023.81743)	11.74873
50	(19907.03483-19941.98752)	(20008.08506-20010.14294)	10.42442
100	(19915.17946-19949.24424)	(20000.12420-20003.17546)	7.32932

Table 3. The Assignment of Nurses in Optimal Sequence in Planning Horizon

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**
1	M	M	M	M	×	M	M	M	M	M	M	×	M	M	M	M	M	M	×	M	M	M	×	M	M	×	M	M	M	M	M
2	M	M	M	M	×	M	M	M	M	M	M	×	M	M	M	M	M	M	×	M	M	M	×	M	M	×	M	M	M	M	M
3	A	N	-	M	M/N	-	M	M	×	A	×	A	×	×	A	A	A	×	N	-	A	×	×	A	N	-	A	×	N	-	M/N
4	×	N	-	A	A	A	N	-	A	A	N	-	A	A	N	-	N	-	N	-	N	-	A	×	A	A	M/A	M/A	×	M	×
5	A	N	-	A	A	A	×	M/A	N	-	×	N	-	M	N	–	M	A	A	A	N	-	M	M/N	-	×	A	M	A	M/A	M/N
6	M	A	M	×	N	-	A	×	×	A	M	M	N	-	N	-	A	N	-	N	-	A	M/N	-	M	M/A	×	A	×	A	N
7	N	-	×	A	N	-	×	A	M/A	N	-	N	-	N	-	A	N	-	N	-	×	M	N	-	×	A	N	-	×	A	A
8	A	A	A	×	A	N	-	A	M/A	M/N	-	N	-	A	M/N	-	M/A	A	×	M/A	×	A	M/A	A	M/A	N	-	A	M	N	-
9	×	M/A	M/A	M/A	A	M/A	M/A	N	-	A	M/A	M/A	M/N	-	M/A	M/A	N	-	M/N	-	M/A	M/A	N	-	M/A	M/N	-	M/N	-	N	-
10	×	A	N	-	A	A	×	N	-	A	N	-	N	-	A	N	-	A	A	A	×	A	M/A	A	N	-	×	×	M/A	×	A
11	A	M	N	-		M/N	-	A	N	-	A	M/A	M/A	M/A	×	N	-	M/A	×	M	N	-	A	M/A	N	-	M/A	N	-	A	M/A
12	×	×	A	N	-	×	A	×	A	N	-	A	M	×	M	A	M	×	M/A	×	M/N	-	A	×	A	A	N	-	N	-	N
13	×	A	N	-	×	A	A	N	-	N	-	A	×	N	-	A	A	N	-	×	×	A	M/N	-	A	N	-	N	×	A	N
14	A	×	N	-	A	N	-	×	N	-	N	-	N	-	N	-	N	-	A	N	-	N	-	N	-	N	-	A	A	×	A
15	M/N	-	M/A	M/A	M/N	-	A	N	-	M	M/A	M	A	M	M/A	M/N	-	M/A	M/A	M/A	M/A	M/N	-	N	-	M/A	M/A	M	M/A	M/N	-
16	M	M/N	-	N	-	-	M	M/N	-	×	A	M	×	×	A	×	M/N	-	M	A	A	N	-	M	N	-	×	A	A	N	-
17	N	-	A	N	-	N	-	A	×	M	A	×	A	N	-	A	N	-	N	-	A	×	N	-	×	M/N	-	N	-	M	×
18	N	-	N	-	A	×	N	-	M/A	N	-	N	-	A	×	N	-	A	M	N	-	N	-	A	×	×	N	-	A	×	A

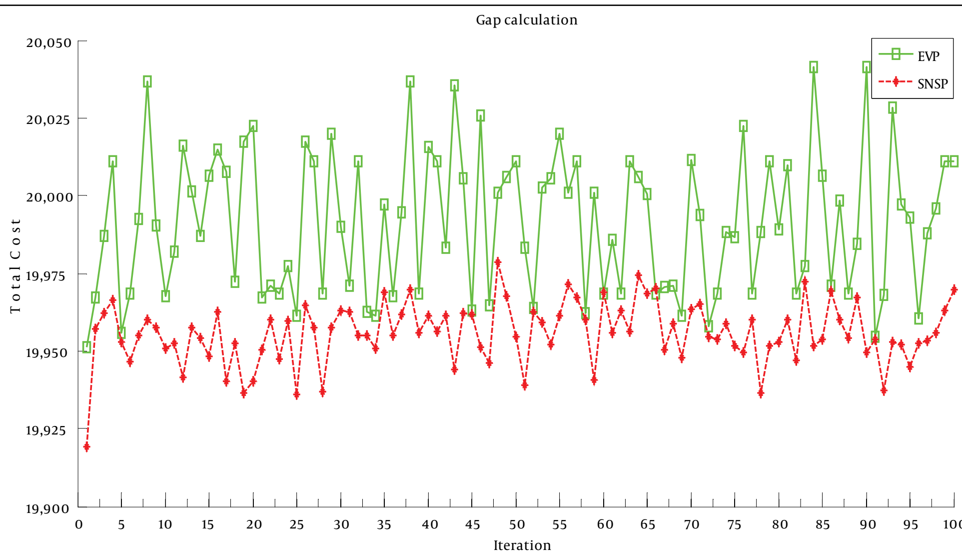

Figure 2. Simulation Results; Total Costs

4. Discussion

This article proposes a stochastic nurse scheduling model for Department of Heart Surgery in Razavi Hospital in an uncertain environment. In this model the demand and stay period of patients are stochastic whose the distribution is determined from historical data. We also consider the priority (vitality) of patients in our model. This priority factor effects the demand for nurses in each shift. Sample Average Approximation (SAA) algorithm is used to solve the proposed stochastic nurse scheduling model and numerical experiments are demonstrated for a model with real data. There are several topics for further research in nurse scheduling problem. First, a stochastic dynamic programming method can be employed for the new nurse scheduling problem in dynamic planning horizon. Second, in application of uncertain parameters, we can use of fuzzy logic and various types of membership functions in nurse scheduling.

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Authors' Contributions

Ali Gholinejad Devin and Mohsen Bagheri: Design of the project. Ali Gholinejad Devin and Azra Izanloo: Writing the manuscript. Mohsen Bagheri and Azra Izanloo: Supervision of the study.

References

1. Warner DM. Scheduling Nursing Personnel According to Nursing Preference: A Mathematical Programming Approach. *Oper Res.* 1976;**24**(5):842–56.
2. Millar HH, Kiragu M. Cyclic and non-cyclic scheduling of 12 h shift nurses by network programming. *Eur J Oper Res.* 1998;**104**(3):582–92.
3. Venkataraman R, Brusco MJ. An integrated analysis of nurse staffing and scheduling policies. *Omega.* 1996;**24**(1):57–71.
4. Ozkarahan I. A flexible nurse scheduling support system. *Comput Methods Programs Biomed.* 1989;**30**(2-3):145–53.
5. Jaumard B, Semet F, Vovor T. A generalized linear programming model for nurse scheduling. *Eur J Oper Res.* 1998;**107**(1):1–18.
6. Klinz B, Pferschy U, Schauer J. ILP Models for a Nurse Scheduling Problem. In: Waldmann KH, Stocker U editors. *Operations Research Proceedings.* Berlin Heidelberg: Springer; 2007. pp. 319–24.
7. Bard JF, Purnomo HW. Short-term nurse scheduling in response to daily fluctuations in supply and demand. *Health Care Manag Sci.* 2005;**8**(4):315–24.
8. Hattori H, Ito T, Ozono T, Shintani T. A Nurse Scheduling System Based on Dynamic Constraint Satisfaction Problem. In: Ali M, Esposito F editors. *Innovations in Applied Artificial Intelligence.* Berlin Heidelberg: Springer; 2005. pp. 799–808.
9. Parr D, Thompson JM. Solving the multi-objective nurse scheduling problem with a weighted cost function. *Ann Oper Res.* 2007;**155**(1):279–88.
10. Punnakitikashem P, Rosenberger JM, Behan DB. Stochastic programming for nurse assignment. *Comput Optim Appl.* 2008;**40**(3):321–49.
11. Fan N, Mujahid S, Zhang J, Georgiev P, Papajorgji P, Steponavice I, et al. The Application of Bayesian Optimization and Classifier Systems in Nurse Scheduling. In: Pardalos PM, Georgiev PG, Papajorgji P, Neugaard B editors. *Systems Analysis Tools for Better Health Care Delivery.* New York: Springer; 2013. pp. 65–98.
12. Li J, Aickelin U. The Application of Bayesian Optimization and Classifier Systems in Nurse Scheduling. In: Bullinaria J, Rowe J, Tino, A. P. , Kaban, A. , Schwefel, H. P. editors. *Parallel Problem Solving from Nature - PPSN VIII* . Springer: Berlin Heidelberg; 2004. pp. 581–90.
13. Topaloglu S, Selim H. Nurse Scheduling Using Fuzzy Multiple Objective Programming. In: Okuno H, Ali M editors. *New Trends in Applied Artificial Intelligence* . Berlin Heidelberg: Springer; 2007. pp. 54–63.
14. Topaloglu S, Selim H. Nurse scheduling using fuzzy modeling approach. *Fuzzy Sets Syst.* 2010;**161**(11):1543–63.
15. Maenhout B, Vanhoucke M. An electromagnetic meta-heuristic for the nurse scheduling problem. *J Heuristics.* 2007;**13**(4):359–85.
16. Landa silva D, Le K. A Simple Evolutionary Algorithm with Self-adaptation for Multi-objective Nurse Scheduling. In: Cotta C, Sevaux M, Sörensen K editors. *Adaptive and Multilevel Metaheuristics.* Berlin Heidelberg: Springer; 2008. pp. 133–55.
17. Tsai CC, Li SHA. A two-stage modeling with genetic algorithms for the nurse scheduling problem. *Expert Syst Appl.* 2009;**36**(5):9506–12.
18. Ohki M, Uneme SY, Kawano H. Effective Mutation Operator and Parallel Processing for Nurse Scheduling. In: Sgurev V, Hadjiski M, Kacprzyk J editors. *Intelligent Systems: From Theory to Practice.* Berlin Heidelberg: Springer; 2010. pp. 229–42.
19. Zhang Z, Hao Z, Huang H. Hybrid Swarm-Based Optimization Algorithm of GA & VNS for Nurse Scheduling Problem. In: Liu B, Chai C editors. *Information Computing and Applications.* Berlin Heidelberg: Springer; 2011. pp. 375–82.
20. Stougie L, Van Der Vlerk MH. *Approximation in stochastic integer programming.* Berlin: University of Groningen; 2003. Available from: http://edoc.hu-berlin.de/docviews/abstract.php?id=26729.
21. Birge JR, Louveaux F. *Introduction to stochastic programming.*: Springer Science & Business Media; 2011.
22. Shapiro A, Philpott A. A tutorial on stochastic programming. *Manuscript.* 2007.
23. Kleywegt AJ, Shapiro A, Homem-de-Mello T. The sample average approximation method for stochastic discrete optimization. *SIAM J Optim.* 2002;**12**(2):479–502.
24. Ahmed S, Alexander S, Shapiro ER. The sample average approximation method for stochastic programs with integer recourse. 2007.
25. Shapiro A, Homem-de-Mello T. On the rate of convergence of optimal solutions of Monte Carlo approximations of stochastic programs. *SIAM J Optim.* 2000;**11**(1):70–86.
26. Mak WK, Morton DP, Wood RK. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Oper Res Lett.* 1999;**24**(1):47–56.